

Exercice 1

$$\cos\left(\frac{p}{4} + x\right) = \cos\frac{p}{4}\cos x - \sin\frac{p}{4}\sin x = \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

$$\sin\left(\frac{p}{4} + x\right) = \sin\frac{p}{4}\cos x + \sin x\cos\frac{p}{4} = \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

Exercice 2

$$\cos\left(\frac{7p}{12}\right) = \cos\left(\frac{p}{4} + \frac{p}{3}\right) = \cos\frac{p}{4}\cos\frac{p}{3} - \sin\frac{p}{4}\sin\frac{p}{3} = \frac{\sqrt{2}}{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$$

De même : $\sin\left(\frac{7p}{12}\right) = \frac{\sqrt{2}}{4}(1 + \sqrt{3})$

$$\cos\left(\frac{p}{12}\right) = \cos\left(-\frac{p}{4} + \frac{p}{3}\right) = \cos\frac{p}{4}\cos\frac{p}{3} + \sin\frac{p}{4}\sin\frac{p}{3} = \frac{\sqrt{2}}{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{4}(1 + \sqrt{3}) \text{ et de}$$

même $\sin\left(\frac{p}{12}\right) = \frac{\sqrt{2}}{4}(-1 + \sqrt{3})$

Exercice 3

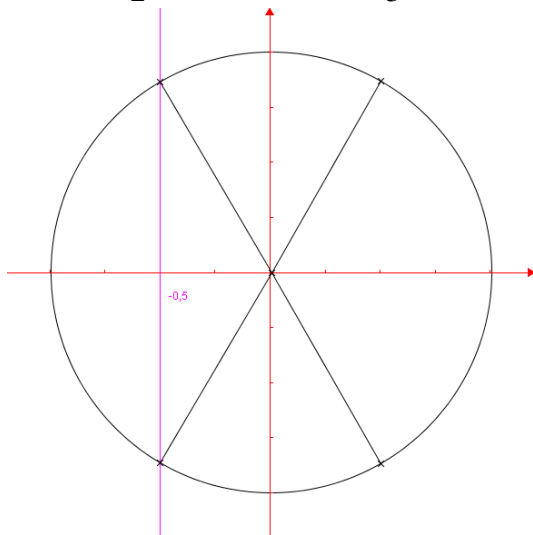
$$\sqrt{2 + 2\cos q} = \sqrt{2 + 2\left(2\cos^2\frac{q}{2} - 1\right)} = \sqrt{4\cos^2\frac{q}{2}} = 2\left|\cos\frac{q}{2}\right| = 2\cos\frac{q}{2} ;$$

$$\sqrt{2 + \sqrt{2 + 2\cos q}} = \sqrt{2 + 2\cos\frac{q}{2}} = \sqrt{2 + 2\left(2\cos^2\frac{q}{4} - 1\right)} = 2\cos\frac{q}{4}$$

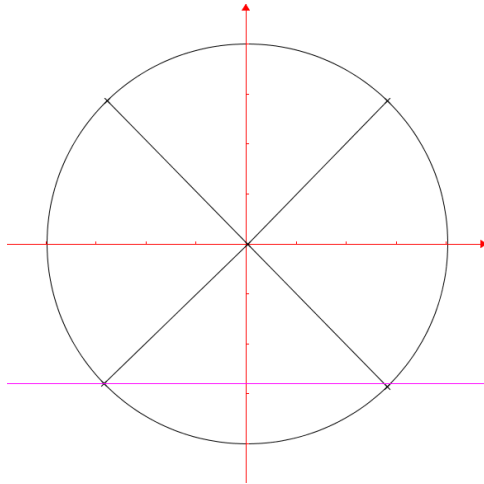
Exercice 4

1) $\cos x = -\frac{1}{2}$: c'est la famille $\frac{p}{3}$, aidons nous d'un cercle trigonométrique :

$$x = \frac{2p}{3} \text{ ou } x = -\frac{2p}{3}$$



2) $\sin x = -\frac{\sqrt{2}}{2}$: c'est la famille $\frac{p}{4}$; aidons nous d'un cercle trigonométrique :



$$x = -\frac{p}{4} \text{ ou } x = -\frac{3p}{4}$$

3) $\cos x = -\frac{1}{2}$ et $\sin x = -\frac{\sqrt{3}}{2}$: c'est la famille $\frac{p}{3}$, aidons nous du cercle trigonométrique de la question 1 : $x = -\frac{2p}{3}$.

4) $\cos x \geq 0$ si $x \in \left[-\frac{p}{2}; \frac{p}{2}\right]$.

Exercice 5

$$S = \sin \frac{p}{3} + \sin \frac{2p}{3} + \sin \frac{4p}{3} + \sin \frac{5p}{3} = \sin \frac{p}{3} + \sin \left(p - \frac{p}{3}\right) + \sin \left(p + \frac{p}{3}\right) + \sin \left(2p - \frac{p}{3}\right) =$$

$$\sin \frac{p}{3} + \sin \frac{p}{3} - \sin \frac{p}{3} - \sin \frac{p}{3} = 0$$

Exercice 6

$$A = \sin \frac{13p}{12} \times \cos \left(-\frac{p}{12}\right) - \sin \frac{p}{12} \times \cos \frac{11p}{12} = \sin \left(p + \frac{p}{12}\right) \times \cos \frac{p}{12} - \sin \frac{p}{12} \times \cos \left(p - \frac{p}{12}\right) =$$

$$-\sin \frac{p}{12} \times \cos \frac{p}{12} - \sin \frac{p}{12} \times \left(-\cos \frac{p}{12}\right) = 0$$